

**Contrôle de Mathématiques - Sujet B - CORRIGE****Question de cours :** / 1 pts

Qu'est-ce qu'un nombre réel ? Les réels sont constitués des nombres rationnels et des nombres irrationnels. Tous les nombres rationnels sont-ils réels ? Par définition, tous les nombres sont des réels, y compris les rationnels.

Citez différents exemples de nombres réels non rationnels :  $\sqrt{2}$ ,  $\sqrt{3}$  et  $\pi$ .

**Exercice 1 :** / 8 pts

$$A = \frac{5}{3} - \frac{3}{4} + \frac{1}{12} = \frac{5 \times 4}{3 \times 4} - \frac{3 \times 3}{4 \times 3} + \frac{1}{12} = \frac{20}{12} - \frac{9}{12} + \frac{1}{12} = \frac{20 - 9 + 1}{12} = \frac{12}{12} = 1 \quad A \in \mathbb{N}$$

$$B = \frac{21}{10} \times \frac{25}{12} \times \frac{14}{15} = \frac{21 \times 25 \times 14}{10 \times 12 \times 15} = \frac{7 \times \boxed{3} \times \boxed{5} \times \boxed{5} \times 7 \times \boxed{2}}{\boxed{5} \times \boxed{2} \times 4 \times 3 \times \boxed{3} \times \boxed{5}} = \frac{7 \times 7}{4 \times 3} = \frac{49}{12} \quad B \in \mathbb{Q}$$

$$C = \frac{\sqrt{49}}{\sqrt{64} + \sqrt{9}} = \frac{7}{8 + 3} = \frac{7}{11} \quad C \in \mathbb{Q}$$

$$D = \frac{3 - 4 \times 5}{2 - \frac{2}{5}} = \frac{3 - 20}{\frac{2 \times 5}{1 \times 5} - \frac{2}{5}} = \frac{-17}{\frac{10}{5} - \frac{2}{5}} = \frac{-17}{\frac{8}{5}} = -17 \times \frac{5}{8} = -\frac{17}{1} \times \frac{5}{8} = -\frac{85}{8} \quad D \in \mathbb{Q}$$

$$E = \frac{2}{7} \times \frac{4}{5} \times \frac{7}{2} = \frac{2}{7} \times \frac{4 \times 3}{5 \times 2} = \frac{2}{7} \times \frac{15}{5 \times \boxed{2}} = \frac{2}{7} \times \frac{15}{12} = \frac{2}{7} \times \left(-\frac{13}{12}\right) \times \frac{12}{5} = -\frac{2 \times 13 \times \boxed{12}}{7 \times \boxed{12} \times 5} = -\frac{26}{35} \quad E \in \mathbb{Q}$$

$$F = 3\sqrt{20} - 4\sqrt{45} + 6\sqrt{500} = 3 \times \sqrt{4} \times \sqrt{5} - 4 \times \sqrt{9} \times \sqrt{5} + 6 \times \sqrt{100} \times \sqrt{5} = 3 \times 2 \times \sqrt{5} - 4 \times 3 \times \sqrt{5} + 6 \times 10 \times \sqrt{5} \\ = 6\sqrt{5} - 12\sqrt{5} + 60\sqrt{5} = (6 - 12 + 60)\sqrt{5} = 54\sqrt{5} \quad F \in \mathbb{R}$$

**Exercice 2 :** / 4 pts

$$A = \frac{10^4 \times 10^6}{(10^6)^2 \times 10^{-11}} = \frac{10^{4+6}}{10^{6 \times 2} \times 10^{-11}} = \frac{10^{10}}{10^{12} \times 10^{-11}} = \frac{10^{10}}{10^{12-11}} = \frac{10^{10}}{10^1} = 10^{10-1} = 10^9$$

$$B = \frac{6^{-1} \times 6^{-7}}{(6^{-4})^{-2} \times 6^{-3}} = \frac{6^{-1+(-7)}}{6^{-4 \times (-2)} \times 6^{-3}} = \frac{6^{-8}}{6^8 \times 6^{-3}} = \frac{6^{-8}}{6^{8+(-3)}} = \frac{6^{-8}}{6^5} = 6^{-8-5} = 6^{-13}$$

$$C = \frac{15 \times 10^{-4} \times 30 \times 10^2}{50 \times 10^{-6}} = \frac{15 \times 30}{50} \times \frac{10^{-4} \times 10^2}{10^{-6}} = \frac{3 \times \boxed{5} \times 3 \times \boxed{10}}{\boxed{5} \times \boxed{10}} \times \frac{10^{-4+2}}{10^{-6}} = 9 \times \frac{10^{-2}}{10^{-6}} = 9 \times 10^{-2-(-6)} \\ = 9 \times 10^{-2+6} = 9 \times 10^4$$

**Exercice 3 :** / 2 pts

$$A = 2 \times 80000 \times 10^{-4} = 2 \times 8 \times 10^4 \times 10^{-4} = 16 \times 10^{4-4} = 16 \times 10^0 = 16 = 1,6 \times 10$$

$$B = 3 \times 0,000004 \times 10^7 = 3 \times 4 \times 10^{-6} \times 10^7 = 12 \times 10^{-6+7} = 12 \times 10^1 = 1,2 \times 10 \times 10 = 1,2 \times 10^2$$

**Exercise 4:** 2,5 pts

<p>A number line from -3 to 4. A blue interval <math>[-3;1]</math> is shown with closed brackets at -3 and 1. A red interval <math>[0;4]</math> is shown with closed brackets at 0 and 4. The intersection is a red interval <math>[0;1]</math> with closed brackets at 0 and 1.</p>	$[-3;1] \cap [0;4] = [0;1]$	$\Leftrightarrow 0 \leq x \leq 1$
<p>A number line from -4 to 2. A blue interval <math>[-4;1]</math> is shown with closed brackets at -4 and 1. A red interval <math>] -3; 2[</math> is shown with open brackets at -3 and 2. The intersection is a red interval <math>] -3; 1[</math> with open brackets at -3 and 1.</p>	$[-4;1] \cap ]-3;2[ = ]-3;1[$	$\Leftrightarrow -3 < x \leq 1$
<p>A number line from -4 to 4. A blue interval <math>] -\infty; 2]</math> is shown with an open bracket at -2 and a closed bracket at 2. A red interval <math>] -2; +\infty[</math> is shown with an open bracket at -2 and a closed bracket at 2. The intersection is a red interval <math>] -2; 2]</math> with an open bracket at -2 and a closed bracket at 2.</p>	$]-\infty;2] \cap ]-2;+\infty[ = ]-2;2]$	$\Leftrightarrow -2 < x \leq 2$
<p>A number line from -3 to 4. A blue interval <math>] -3; 1[</math> is shown with open brackets at -3 and 1. A red interval <math>[1;4]</math> is shown with closed brackets at 1 and 4. There is no overlap between the intervals.</p>	$]-3;1[ \cap [1;4] = \emptyset$	$\Leftrightarrow x \in \emptyset$
<p>A number line from -6 to 2. A blue interval <math>[-6;3]</math> is shown with closed brackets at -6 and 3. A red interval <math>[-3;2]</math> is shown with closed brackets at -3 and 2. A green interval <math>[-4;3]</math> is shown with closed brackets at -4 and 3. The intersection of all three is a green interval <math>[-6;3]</math> with closed brackets at -6 and 3.</p>	$[-6;3] \cap [-3;2] \cap [-4;3] = [-6;3]$	$\Leftrightarrow -6 \leq x \leq 3$

**Exercise 5:** 2,5 pts

<p>A number line from -3 to 5. A blue interval <math>[-3;3]</math> is shown with closed brackets at -3 and 3. A red interval <math>[1;5]</math> is shown with closed brackets at 1 and 5. The union is a red interval <math>[-3;5]</math> with closed brackets at -3 and 5.</p>	$[-3;3] \cup [1;5] = [-3;5]$	$\Leftrightarrow -3 \leq x \leq 5$
<p>A number line from -3 to 5. A blue interval <math>[-3;4]</math> is shown with closed brackets at -3 and 4. A red interval <math>] -3; 5[</math> is shown with open brackets at -3 and 5. The union is a red interval <math>[-3;5[</math> with a closed bracket at -3 and an open bracket at 5.</p>	$[-3;4] \cup ]-3;5[ = [-3;5[$	$\Leftrightarrow -3 \leq x < 5$
<p>A number line from -4 to 3. A blue interval <math>] -\infty; 1]</math> is shown with an open bracket at -2 and a closed bracket at 1. A red interval <math>] -2; +\infty[</math> is shown with an open bracket at -2 and a closed bracket at 1. The union covers the entire number line.</p>	$]-\infty;1] \cup ]-2;+\infty[ = \mathbb{R}$	$x \in \mathbb{R}$
<p>A number line from -1 to 5. A blue interval <math>] -1; 4[</math> is shown with open brackets at -1 and 4. A red interval <math>[4;5]</math> is shown with closed brackets at 4 and 5. The union is a red interval <math>] -1; 5]</math> with an open bracket at -1 and a closed bracket at 5.</p>	$]-1;4[ \cup [4;5] = ]-1;5]$	$\Leftrightarrow -1 < x \leq 5$
<p>A number line from -5 to 6. A blue interval <math>[-5;1]</math> is shown with closed brackets at -5 and 1. A red interval <math>[1;2]</math> is shown with closed brackets at 1 and 2. A green interval <math>[2;6]</math> is shown with closed brackets at 2 and 6. The union is a green interval <math>[-5;6[</math> with a closed bracket at -5 and an open bracket at 6.</p>	$[-5;1] \cup [1;2] \cup [2;6] = [-5;6[$	$\Leftrightarrow -5 \leq x < 6$